SOLITARY CYLINDRICAL LAYER OF ELECTRICALLY CONDUCTING MAGNETIC FLUID

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The possibility of the existence and stability conditions for a solitary layer of electrically conducting magnetic fluid not resting on solid walls is studied.

The formation of a stationary, solitary, i.e., not resting on a wall, cylindrical fluid layer (Fig. 1) in a homogeneous gas atmosphere is impossible in the hydromechanics of non-electrically conductive, nonmagnetic fluids. This is related to the fact that under weightlessness the single forces acting on the fluid layer – the surface tension forces – have no component equilibrated and directed to the center and drag the layer to the center. A cylindrical layer can be produced only by utilizing the external gas pressure drop, but such a layer, as is later shown, is unstable with respect to the simplest disturbances. The possibility of the existence of a stationary layer appears for an electrically conductive magnetic fluid (EMF) through which an electrical current passes and which surrounds a coaxial cylindrical solid conductor with current since the radially directed mass forces of electromagnetic origin that occur equilibrate the surface tension forces.

Exact analytic solutions of the statics equations for an EMF are found below. In particular, a solution exists that describes a solitary EMF layer. The solutions obtained are investigated as to stability relative to the simplest axisymmetric disturbances.

Let us formulate mathematically the problem of cylindrical EMF layers (see Fig. 2). The cylindrical solid conductor and EMF layers are arranged coaxially under weightlessness. A current J_0 is transmitted along the central solid conductor and a current $J_1 > 0$ is transmitted along the EMF layer. The radius of the central conductor is a_0 and for the EMF layer, the inner radius is a and the outer radius is b, $a_0 \leqslant a \leqslant b$. The behavior of the EMF is described by the system of equations [1]

$$\rho \frac{d\mathbf{v}}{dt} = \eta\Delta\mathbf{v} - \nabla p + \mathbf{f}, \quad \mathbf{f} = \mu_0 M \nabla H + [\mathbf{j}\times\mathbf{B}], \quad \operatorname{div}\mathbf{v} = 0,$$

$$\frac{\partial\mathbf{H}}{\partial t} - \operatorname{rot}[\mathbf{v}\times\mathbf{H}] = \eta_M\Delta\mathbf{H}, \quad \operatorname{div}\mathbf{B} = 0, \quad \mathbf{B} = \mu_0\mu\mathbf{H} = \mu_0(\mathbf{H}+\mathbf{M}), \ \mathbf{M} = \chi\mathbf{H}, \tag{1}$$

$$\operatorname{rot}\mathbf{H} = \mathbf{j}, \quad \rho c_p \frac{dT}{dt} = \lambda\Delta T + \frac{\eta}{2}\sum_{i,k=1}^{3}(\nabla_i v_k + \nabla_k v_i)^2 + \sigma_C^{-1} j^2,$$

on the EMF boundaries (r = a, b)

$$\{p\} = \sigma(K_1 + K_2) - \mu_0(\mathbf{M}\mathbf{n})^2/2,$$
$$\{\mathbf{H}\}\times\mathbf{n} = 0, \quad \{\mathbf{B}\}\mathbf{n} = 0, \quad p_e|_{r=a-0} = p_1, \quad p_e|_{r=b+0} = p_2, \tag{2}$$

at infinity $H|_{r\to\infty} = 0$, where $\{A\} = A - A_e$ is the jump in the quantity A in the fluid and in the surrounding gas (A_e) on the EMF surface, **B** is the magnetic field induction, and $\eta_M = 1/(\mu_0\mu\sigma_C)$ is the diffusion coefficient of the magnetic field.

For a fixed EMF (**v** = 0), under the assumption of constancy of the coefficients σ, μ, ρ, λ, c_p, σ_C, the system (1), (2) has the stationary solution

$$\mathbf{H} = (H_r;\ H_\theta;\ H_z) = \left(0;\ \frac{J_0}{2\pi r} + \frac{J_1}{2\pi(b^2-a^2)}\left(r - \frac{a^2}{r}\right);\ 0\right),$$

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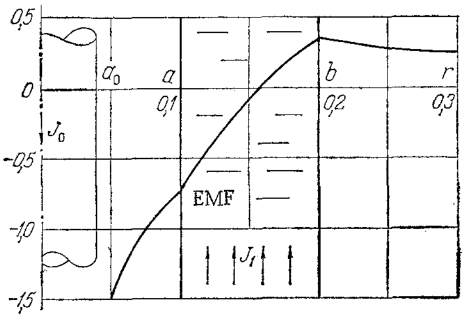

Fig. 1. Diagram of the location of a solitary EMF layer and graph of the dependence of the dimensionless magnetic field intensity H (3π/2) (0.1 m/20 A), on the radius r (m) for a = 0.1 m, b = 0.2 m, J_0 = −20 A, J_1 = 40 A.

$$p=\frac{p_1+p_2}{2}+\frac{\sigma}{2}\left(\frac{1}{b}-\frac{1}{a}\right)+\frac{\alpha}{4}\left(\frac{1}{a^2}+\frac{1}{b^2}-\frac{2}{r^2}\right)$$
$$+\frac{\beta}{2}\ln\left(\frac{r^2}{ab}\right)+\frac{\gamma}{4}(2r^2-a^2-b^2),\quad r\in[a,\ b], \tag{3}$$

with the mass force distribution within the EMF

$$f=\alpha/r^3+\beta/r+\gamma r,\quad \mathbf{f}=(f;\ 0;\ 0),$$
$$\alpha=-\mu_0(\mu-1)A^2\leqslant 0,\quad \beta=-j_1\mu_0\mu A,\quad \gamma=-\mu_0(\mu+1)j_1^2/4\leqslant 0,$$
$$A=\frac{1}{2\pi}\left(J_0-\frac{a^2J_1}{b^2-a^2}\right),\quad j_1=\frac{J_1}{\pi(b^2-a^2)}.$$

Because of heating by currents a layer element of length dz liberates heat of an intensity

$$Q=dzJ_0^2/(\sigma_0\pi a_0^2)+dzJ_1^2/(\sigma_1\pi(b^2-a^2)),$$

which is either expended in evaporation in the steady regime or is liberated into the surrounding space.

If the surrounding external walls have a constant temperature T_0, then in the absence of convection ($\mathbf{v}$ = 0) and evaporation at the temperature, the boundary conditions

$$T|_{r=b_0-0}=T_0,$$
$$\left.\frac{\partial T}{\partial r}\right|_{r=\rho}=-q=-Q/(2\pi\rho\lambda dz),\quad \rho\geqslant b,$$

are given, and the solution (3) allows a stationary continuous temperature distribution

$$T=b_0q_0\ln(b_0/r)+T_0,\quad q_0=q|_{\rho=b_0},\quad b<r<b_0,$$
$$T=b(q_b-Cb/2)\ln(b/r)+T_b+Cb^2/4,$$
$$C=j^2/(\sigma_1\lambda),\quad q_b=q/_{\rho=b},$$
$$T_b=T|_{r=b}=b_0q_0\ln(b_0/b)+T_0,\quad a<r<b.$$

The solution (3) can describe a cylindrical fluid layer both resting on a wall (r = a_0, b_0) and no resting thereon ($a_0 < a < b_0$), a solitary layer.

This last case is most interesting since there is no direct analog in "ordinary," i.e., nonelectrically conducting, nonmagnetic, fluids. In these nonelectrically conducting, nonmagnetic fluids (i.e., J_1 = 0, μ = 1) there are no mass forces ($\mathbf{f}$ = 0), and under conditions of homogeneity of the gas pressure ($p_1 = p_2$) a solitary stationary layer is impossible. For an EMF (J_1 = 0, $\mu \neq 1$) the possibility occurs of the existence of a solitary layer retained far from the walls by mass forces $\mathbf{f}$ in a homogeneous gas atmosphere ($p_1 = p_2$). This holds if and only if a level r = r_0($a < r_0 <$ b) exists within the EMF layer $r\in[a, b]$, at which there is no mass force $\mathbf{f}$ applied to the EMF element ($f|_{r=r_0} = 0$), and the forces $\mathbf{f}$ acting on both sides of the layer ($r < r_0$, $r > r_0$) are directed differently and in sum are equivalent to the surface tension force.

For $\mu \neq 1$ two solutions (3) can exist for a magnetic fluid for a stationary solitary cylindrical EMF layer within which are the levels

$$r_i = a\sqrt{\frac{\mu-1}{\mu\mp1}\left[1-\left(\frac{b^2}{a^2}-1\right)\frac{J_0}{J_1}\right]},\quad i=1,\ 2$$

(the + sign for r_1 and the − sign for r_2) at which the fluid does not experience the action of the mass forces $f|_{r=r_i} = 0$. The first solution (near $r = r_1$) exists under the condition $(\mu-1)\frac{J_0}{J_1}\left(\frac{b^2}{a^2}-1\right)<-2$, and the second for $J_0 < 0$.

For $\mu = 1$ (a nonmagnetic conducting fluid), there is either one such solution of a solitary fluid layer (under the condition $J_0 < 0$) or (otherwise for $J_0 > 0$) there is no such solution ($f = 0$) and then only a fluid layer pressed to the inner cylinder exists $f < 0$, $a_0 = a$, for constant external gas pressure ($p_1 = p_2$).

There is the question of the stability of the stationary solutions obtained for the equilibrium positions of the EMF relative to perturbations of different kinds. The first solution is always unstable relative to axisymmetric perturbations disturbing the continuity of the layer: $f < 0$ for $a < r < r_1$, $f > 0$ for $r_1 < r < b$; and for constant pressure in the gas the mass force tries to break up the EMF layer at the level $r = r_1$ to different sides. Later, however, the decomposition of this layer occurs by another means, more energetically suitable, and the developing instability is a direct analog of the Rayleigh–Taylor instability in ordinary hydrodynamics. The second solution is table relative to rupturing disturbances ($f < 0$ for $r_2 < r < b$, $f > 0$ for $a < r < r_2$). The single solution for $\mu = 1$ for a nonmagnetic conducting fluid is also stable relative to these perturbations.

Let us investigate the stability of the stationary solution (3) (a, b = const) of an EMF relative to small axisymmetric disturbances of a cylindrical layer surface of the form

$$a = a(t) = a|_{t=0} + a'\exp(-i\omega t),\quad b = b|_{t=0} + b'\exp(-i\omega t) \tag{4}$$

under the condition of conservation of the section area

$$S = \pi(b^2 - a^2) = \text{const},\quad J_1 = \text{const}. \tag{5}$$

Substituting (4), (5) into (1), (2) in a linear approximation in the amplitudes results in a dispersion relationship

$$\omega^2\rho\ln(b/a) = -\sigma(1/a^3 + 1/b^3) - (1/a)(dF/da),\quad F = \int\limits_{a(t)}^{b(t)} f\,dr, \tag{6}$$

from which it follows that the solution (3) is stable relative to the disturbances (4), (5) for

$$(1/a)(dF/da) + \sigma(1/a^3 + 1/b^3) < 0. \tag{7}$$

This condition reduces to the condition for well conducting MF ($\eta_M \to 0$)

$$\begin{aligned}&\mu[A^2((a+d)^{-4}-a^{-4}) + Aj_1((a+d)^{-2}-a^{-2})] >\\ &> A^2((a+d)^{-4}-a^{-4}) + \sigma((a+d)^{-3}+a^{-3})/\mu_0,\quad d = b-a,\end{aligned} \tag{8}$$

and for weakly conducting MF ($\eta_M \to \infty$)

$$\begin{aligned}&\mu[A^2((a+d)^{-4}-a^{-4}) + 2Aj_1((a+d)^{-2}-a^{-2}) + j_1^2\ln(a/b)] >\\ &> A^2((a+d)^{-4}-a^{-4}) + Aj_1((a+d)^{-2}-a^{-2}) + \sigma((a+d)^{-3}+a^{-3})/\mu_0.\end{aligned} \tag{9}$$

The form of the stability conditions (8) and (9) shows that, in contrast to a nonmagnetic, nonelectrically conducting fluid, the presence of the factor $\mu > 1$ in the left side for an EMF broadens the range of physical parameters for which a solitary layer exists that is stable relative to the disturbances investigated (4), and the stronger the magnetic properties of the EMF, i.e., the higher the possible magnetic permittivity μ, the broader this range of allowable physical parameters. In particular, it follows from conditions (8) and (9) that a solitary layer of nonmagnetic, nonelectrically conducting fluid is unstable relative to the disturbances (4) for any values of the gas pressures p_1, p_2.

The current magnitudes enter the stability conditions (8), (9) of a solitary layer only in the form of the ratio J_1/J_0. Two contradictory requirements are additionally imposed on the absolute values of the currents J_0, J_1:

1) The current should be sufficiently high so that the mass magnetic forces would overcome the surface forces directed to the center;

2) the currents should be sufficiently constrained so that their heat liberation would not alter the aggregate state of the EMF and would not produce thermal stresses and thermal convection hindering the stability of the layer as a whole.

Because of (3) or (6) the first condition can be estimated approximately as

$$\mu_0\mu J_0 J_1/a \gtrsim \sigma.$$

In particular, it hence follows as in the stability conditions (8) and (9) that the higher the permittivity μ, the smaller the current needed for the existence of a solitary EMF layer.

For known highly stable EMF [2-5] these two contradictory requirements on the absolute value of the currents are satisfied simultaneously for a certain range of the physical parameters. Estimates show that for currents of $J_0 \sim 100$ A and $J_1 \sim 8$ A an EMF on the basis of mercury with admixtures of tin, bismuth, and lithium [2] ($\sigma \sim 0.3$ N/m, $1/\sigma_1 \sim 5 \cdot 10^{-7}$ $\Omega \cdot$m) is heated to completely acceptable temperatures $T \sim 200$°C. The boundedness of the range of allowable currents is explained by the fact that existing EMF [2-5] are low-concentrations (up to 4% by weight) of quite fine particles of approximately up to 50 Å in size, and therefore, with quite low μ. Production of an EMF with particle concentrations and sizes such as in nonelectrically conducting EMF would substantially lower the minimal magnitude of the currents needed for the existence of a solitary layer.

NOTATION

T, temperature; J_0, J_1, magnitudes of the electrical currents flowing in the solid conductor and in the EMF; a_0, a, b, radius of the solid conductor, the inner and outer radii of the EMF layer; p, pressure; v, **M**, ρ, EMF velocity, magnetization, and density; **B**, **H**, magnetic field induction and intensity; μ_0, magnetic permittivity of a vacuum (magnetic constant); K_1, K_2, curvatures of the principal normal sections of the surface; σ, σ_C, c_p, η_M, λ, EMF surface tension, conductivity, specific heat, magnetic field diffusion, heat-conductivity coefficients, and Q is the specific power of the heat liberation.

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